
# Database Tuples Play Cooperative Games

**Ester Livshits**

Joint work with:
Leopoldo Bertossi, Benny Kimelfeld, Alon Reshef, Moshe Sebag

| AUTHOR | |
|---|---|
| **Name** | **Affiliation** |
| Alice | UCLA |
| Bob | NYU |
| Cathy | MIT |
| David | UCSD |
| Ellen | NYU |

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| Alice | A |
| Alice | B |
| Bob | C |
| Cathy | C |
| Cathy | D |
| David | C |

| CITATIONS | |
|---|---|
| **PAPER** | **CITS** |
| A | 18 |
| B | 2 |
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$$q(z, w): -\text{AUTHOR}(x, y), \text{INSTITUE}(y, 'CA'), \text{PUBLICATION}(x, z), \text{CITATIONS}(z, w)$$

**Why we obtained a particular answer?**

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**Why we did not obtain some other answer?**

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**Why we obtained a particular answer?**

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**Why we did not obtain some other answer?**

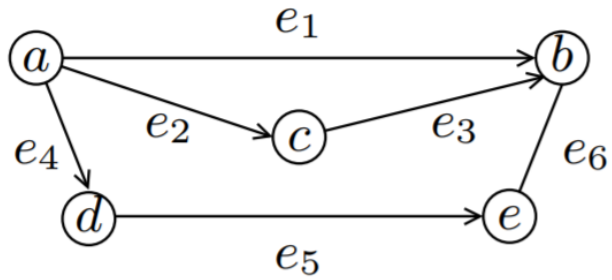
# Which tuples in the database explain the query result?

# Measuring Contribution

- Causal responsibility [Meliou et al. 2010]
  - $t$ is a **counterfactual cause** for $q$ if $D \vDash q$ and $D \setminus \{t\} \nvDash q$
  - $t$ is an **actual cause** for $q$ if $D \setminus \Gamma \vDash q$ and $D \setminus \{\Gamma \cup \{t\}\} \nvDash q$ for some $\Gamma \subseteq D \setminus \{t\}$
  - The responsibility of $t$ is $\frac{1}{1+|\Gamma_{\min}|}$

Contingency set

- Not extendable to aggregate queries
- May be counterintuitive




Is there a path from a to b?

# Measuring Contribution

- Causal effect [Salimi et al. 2016]
  - See the database as a probabilistic database
  - $\mathrm{CE}(t) = \mathrm{E}(q \mid t \in D) - \mathrm{E}(q \mid t \notin D)$

**What makes the choice of a contribution score a good one?**

# Shapley Value

- A widely known profit-sharing formula in cooperative game theory
- Introduced by Lloyd Shapley in 1953
- Applied in various areas beyond cooperative game theory:
  - Pollution responsibility in environmental management
  - Influence measurement in social network analysis
  - Identifying candidate autism genes
  - Bargaining foundations in economics
  - Takeover corporate rights in law
  - Explanations in machine learning

# Shapley Value

Set $A$ of players:

How to distribute the total wealth among the players?

**Machine learning**
[Lundberg, Lee 2017]

Features

Prediction

**Query answering**
[L et al. 2020]

Tuples

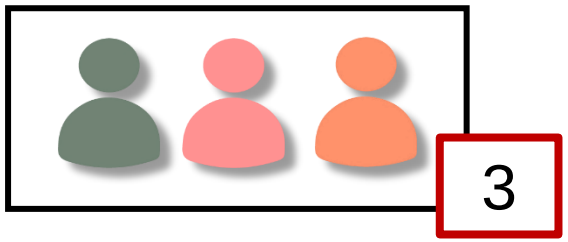
Answer

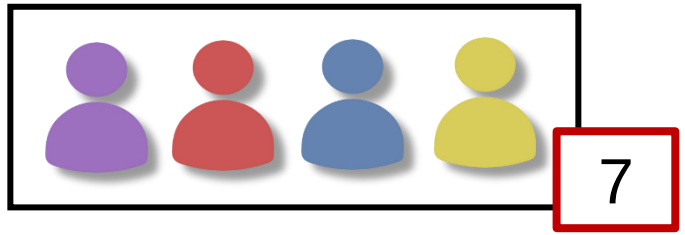
**Inconsistency**
[L, Kimelfeld 2021]

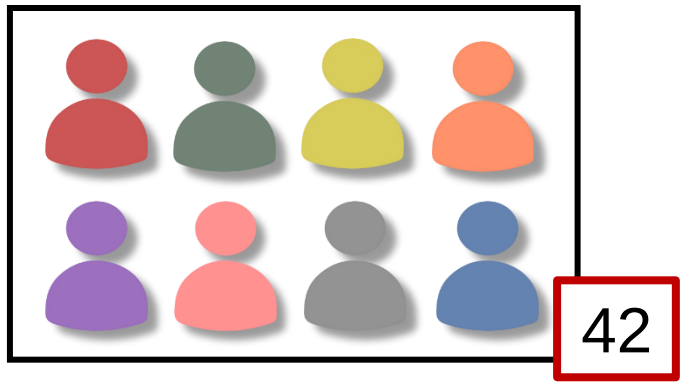
Tuples

Measure

Wealth function $v: \mathcal{P}(A) \rightarrow \mathbb{R}$:

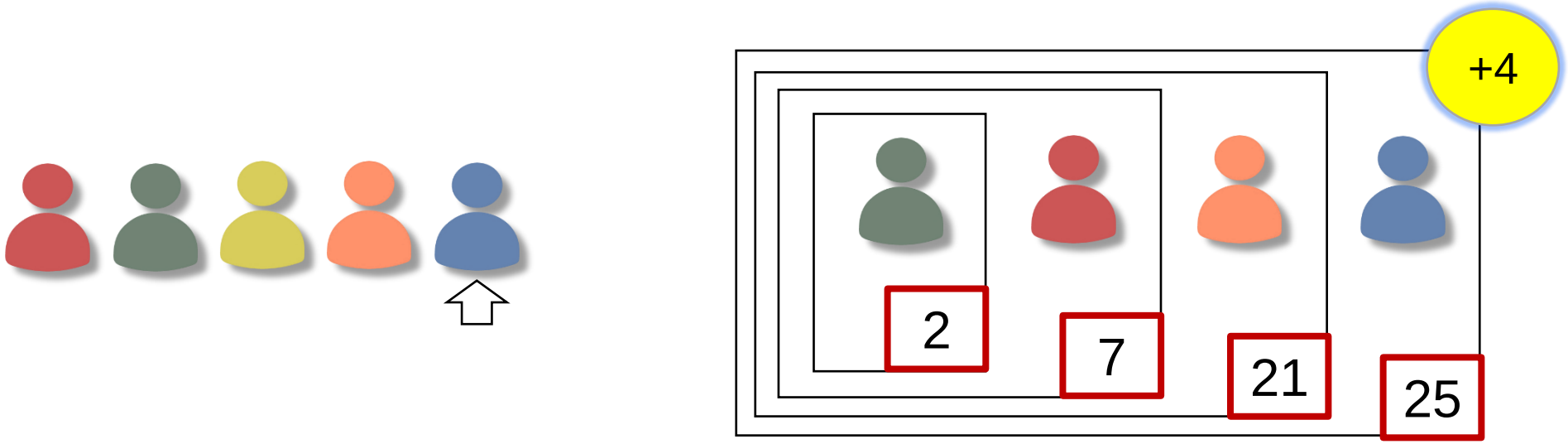


# Shapley Value

$$\text{Shapley}(A, v, a) = \sum_{B \subseteq A \setminus \{a\}} \frac{|B|!\,(|A| - |B| - 1)!}{|A|!} \big(v(B \cup \{a\}) - v(B)\big)$$




The Shapley value is the expected delta due to the addition in a random permutation

# Shapley Value for Database Queries

- Which tuples in the database explain the query result?

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$$SV(Alice) = 20$$
$$SV(Cathy) = 14.67$$
$$SV(Bob) = 2.67$$
$$SV(David) = 2.67$$
$$SV(Ellen) = 0$$

$$q(z,w) :- \text{AUTHOR}(x,y), \text{PUBLICATION}(x,z), \text{CITATIONS}(z,w)$$

$$\text{SUM}_w\langle q(z,w)\rangle$$

← Wealth function

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$$q() : -\text{AUTHOR}(x, y), \text{INSTITUE}(y, 'CA'), \text{PUBLICATION}(x, 'A'), \text{CITATIONS}('A', 18)$$

# Outline

- Explaining Query Answers
- **Computational Complexity**
- Responsibility to Inconsistency

$q() :- \text{AUTHOR}(x, y), \text{INSTITUTE}(y, 'CA'), \text{PUBLICATION}(x, z)$

# Computational Complexity

| Query | Hierarchical | Non-hierarchical |
|---|---|---|
| SJFCQ | PTIME | $\text{FP}^{\#\text{P}}$-complete |
| SJFCQ with negations | PTIME | $\text{FP}^{\#\text{P}}$-complete |
| sum \ count | PTIME | $\text{FP}^{\#\text{P}}$-complete |

[L et al. ICDT 2020]

[Reshef et al. PODS 2020]

- A CQ $q$ is **hierarchical** if for every two existential variables $x$ and $y$:
  - $Atoms(x) \subseteq Atoms(y)$ or
  - $Atoms(y) \subseteq Atoms(x)$ or
  - $Atoms(x) \cap Atoms(y) = \emptyset$

$q_1() :- R(x, y), S(x, z)$

$y$ $z$

$x$

$q() :- \text{AUTHOR}(x, y), \text{INSTITUTE}(y, 'CA'), \text{PUBLICATION}(x, z)$

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$q_2() :- R(x), S(x, y), T(y)$ ✗

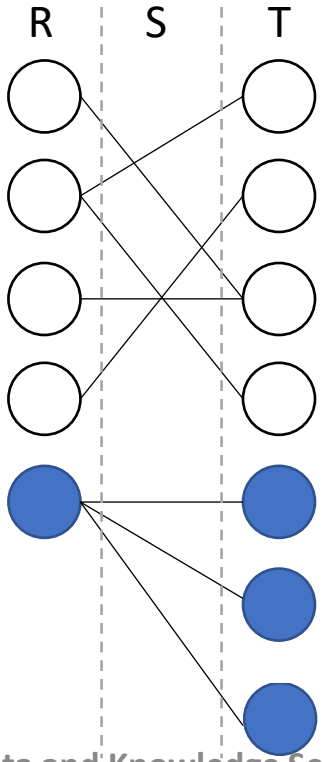
$x$ $y$

# Conjunctive Queries

- To prove hardness, we consider the simplest non-hierarchical query

$$q_{RST}() :- R(x), S(x, y), T(y)$$

- Reduction from counting independent sets in a bipartite graph



# Conjunctive Queries

- Each instance provides us with an equation over $|\mathrm{IS}(g,k)|$
- $|\mathrm{IS}(g,k)|$ - number of independent sets of size $k$ in $g$

$$\begin{pmatrix} 0!(n+1)! & 1!n! & \dots & n!1! \\ 0!(n+2)! & 1!(n+1)! & \dots & n!2! \\ \vdots & \vdots & \vdots & \vdots \\ 0!(2n+1)! & 1!(2n)! & \dots & n!(n+1)! \end{pmatrix} \begin{pmatrix} |\mathsf{IS}(g,0)| \\ |\mathsf{IS}(g,1)| \\ \vdots \\ |\mathsf{IS}(g,n)| \end{pmatrix} = \begin{pmatrix} (n+2)!S_1 - c_1 v_0 \\ (n+3)!S_2 - c_2 v_0 \\ \vdots \\ (2n+2)!S_{n+1} - c_{n+1} v_0 \end{pmatrix}$$

$q() : -\text{AUTHOR}(x, y), \neg\text{INSTITUTE}(y, 'CA'), \text{PUBLICATION}(x, z)$

# Computational Complexity

| Query | Hierarchical | Non-hierarchical |
|---|---|---|
| SJFCQ | PTIME | $\text{FP}^{\#\text{P}}$-complete |
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[L et al. ICDT 2020]

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- A CQ $q$ is **hierarchical** if for every two existential variables $x$ and $y$:
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$\text{SUM}_w\langle q(z, w)\rangle$

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$q(z, w)$: $-\text{AUTHOR}(x, y), \text{PUBLICATION}(x, z), \text{CITATIONS}(z, w)$
$\text{MAX}_w\langle q(z, w)\rangle, \text{MIN}_w\langle q(z, w)\rangle, \text{AVERAGE}_w\langle q(z, w)\rangle$

# Computational Complexity

| Query | Hierarchical | Non-hierarchical |
|---|---|---|
| SJFCQ | PTIME | $\text{FP}^{\#\text{P}}$-complete |
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Hardness can be extended to general numerical queries

- A CQ $q$ is **hierarchical** if for every two existential variables $x$ and $y$:
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# Approximation Complexity

- Computing the Shapley value is often hard
- The picture is more positive when allowing approximation

| Query | Hierarchical | Non-hierarchical |
|---|---|---|
| SJFCQ | PTIME | FPRAS |
| sum \ count | PTIME | FPRAS |

$$\Pr\left[\frac{f(x)}{1+\epsilon} \leq A(x,\epsilon,\delta) \leq (1+\epsilon)f(x)\right] \geq 1-\delta$$

- Generalizes to unions of CQs

# Approximation Complexity

- Additive approximation via Monte Carlo sampling

$$\Pr[f(x) - \epsilon \leq A(x, \epsilon, \delta) \leq f(x) + \epsilon] \geq 1 - \delta$$

- Also a multiplicative approximation due to the "gap property"

For every tuple $t$ in the database $D$:
Shapley($t$)=0 or Shapley($t$)$\geq \frac{1}{p(|D|)}$

- Does not hold when allowing negation
- Negation **fundamentally changes** the complexity picture!

# Approximation Complexity

- With negation, the contribution can be negative

| Student |
|---|
| Name |
| Alice |
| Bob |
| Cathy |
| David |

| TA |
|---|
| Name |
| Alice |
| Bob |
| David |

| Register | |
|---|---|
| Student | Course |
| Alice | OS |
| Alice | AI |
| Bob | OS |
| Cathy | DB |
| Cathy | IC |

$$q() : -\text{Student}(x), \neg\text{TA}(x), \text{Register}(x, y)$$

In some cases, deciding whether Shapley($t$)≠0 is hard

# Banzhaf Power Index

- Causal effect [Salimi et al. 2016]
  - See the database as a probabilistic database
  - $\mathrm{CE}(t) = \mathrm{E}(q \mid t \in D) - \mathrm{E}(q \mid t \notin D)$
- Coincides with the Banzhaf Power Index [Banzhaf 1965]
- Our complexity results extend to this measure

# Outline

- Explaining Query Answers
- Computational Complexity
- **Responsibility to Inconsistency**

# Inconsistent Databases

- A database is inconsistent if it violates integrity constraints


Marion Jones

dbo:height

- 1.524
- 1.778

Cullen Douglas

dbo:birthPlace

- dbr:California
- dbr:Florida

Irene Tedrow

dbo:deathPlace

- dbr:California
- dbr:Hollywood,_Los_Angeles
- dbr:New_York_City

# Inconsistent Databases

- Imprecise data sources
  - Crowd, Web pages, social encyclopedias, sensors, …
- Imprecise data generation
  - natural-language processing, sensor/signal processing, image recognition, …
- Conflicts in data integration
  - Crowd + enterprise data + KB + Web + ...
- Data staleness
  - Entities change address, status, ...
- And so on…

Idea:

# Quantify the extent to which integrity constraints are violated

## Reliability estimation

How reliable is a new data source?

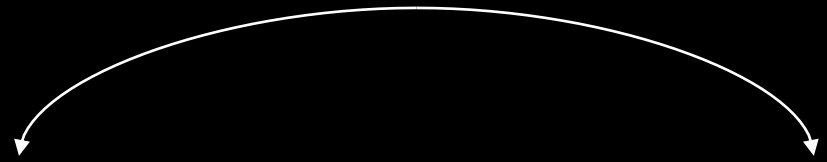
## Progress indication

Progress bar for data repairing


## Action prioritization

Which tuples are mostly responsible for inconsistency?

# How can we quantify the responsibility of individual tuples to inconsistency?



Inconsistency measure

Responsibility sharing mechanism

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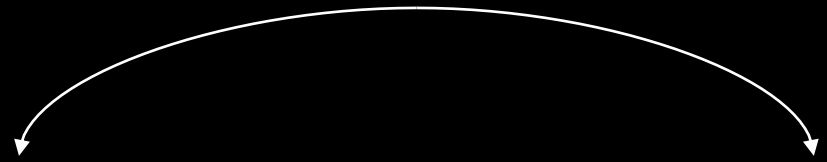
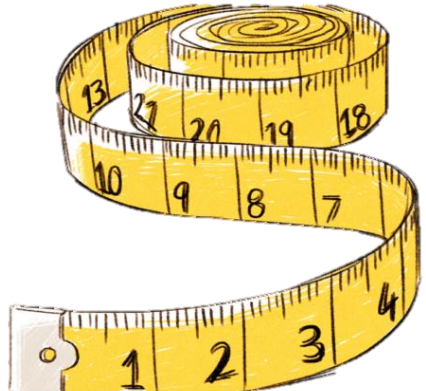


**Inconsistency measure**

Responsibility sharing mechanism

# How to Measure Inconsistency?

- Several measures proposed by the KR and DB communities
  - The drastic measure – 1 if inconsistent, 0 otherwise [Thimm 2017]
  - #minimal inconsistent subsets [Hunter and Konieczny 2008]
  - #problematic tuples [Grant and Hunter 2011]
  - Minimal #tuples to remove to satisfy the constraints [Grant and Hunter 2013], [Bertossi 2018]
  - #maximal consistent subsets [Grant and Hunter 2011]
- What makes a measure a good one? [**L** et al. SIGMOD 2021]



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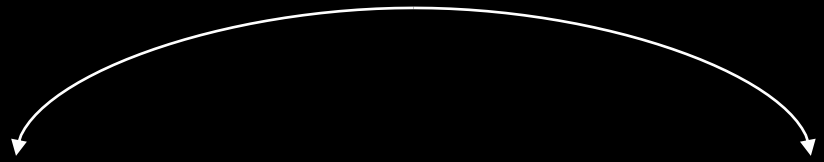

Inconsistency measure

**Responsibility sharing mechanism**

**Shapley Value**

FD: birthCity → birthState

# Computational Complexity

| Measure | lhs chain | No lhs chain, tractable c-repair | other |
|---|---|---|---|
| drastic | PTIME | $\mathrm{FP}^{\#\mathrm{P}}$-complete | |
| #min-inconsistent | PTIME | | |
| #problematic tuples | PTIME | | |
| cardinality repair | PTIME | Open | NP-hard |
| #repairs | PTIME | $\mathrm{FP}^{\#\mathrm{P}}$-complete | |

# Tractable Measures

- $I_{MI}$ - Number of minimal inconsistent subsets

| | Train | Departs | Arrives | Time | Duration |
|---|---|---|---|---|---|
| $f_1$ | 16 | NYP | BBY | 1030 | 315 |
| $f_2$ | 16 | NYP | PVD | 1030 | 250 |
| $f_3$ | 16 | PHL | WIL | 1030 | 20 |
| $f_4$ | 16 | PHL | BAL | 1030 | 70 |
| $f_5$ | 16 | PHL | WAS | 1030 | 120 |
| $f_6$ | 16 | BBY | PHL | 1030 | 260 |
| $f_7$ | 16 | BBY | NYP | 1030 | 260 |
| $f_8$ | 16 | BBY | WAS | 1030 | 420 |
| $f_9$ | 16 | WAS | PVD | 1030 | 390 |


Train Time → Departs

Train Time Duration → Arrives

# Tractable Measures

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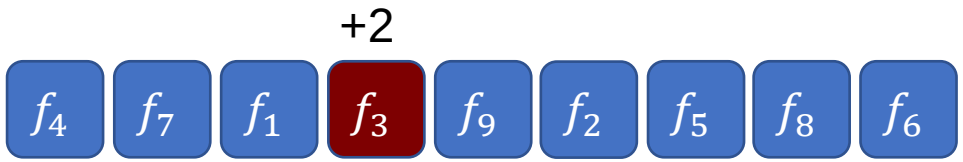


$f$ increases the value of $I_{MI}$ by $k$ if $k$ of the previous tuples conflict with it

# Tractable Measures

- $I_P$ - Number of problematic tuples

| | Train | Departs | Arrives | Time | Duration |
|---|---|---|---|---|---|
| $f_1$ | 16 | NYP | BBY | 1030 | 315 |
| $f_2$ | 16 | NYP | PVD | 1030 | 250 |
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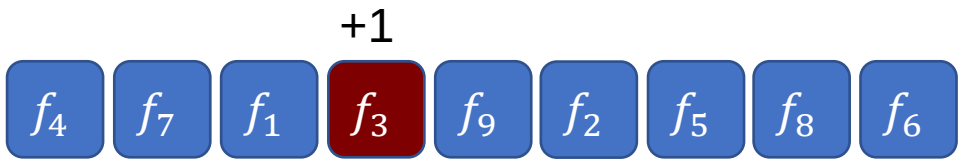
Train Time Duration → Arrives




$f$ increases the value of $I_p$ by $k$ if $(k-1)$ of the previous tuples:

*(1)* conflict with $f$,

*(2)* do not conflict with other tuples that occur before $f$.

# Computational Complexity

| Measure | lhs chain | No lhs chain, tractable c-repair | other |
|---|---|---|---|
| drastic | PTIME | $\mathrm{FP}^{\#\mathrm{P}}$-complete | |
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$\{\boldsymbol{B} \to A, \boldsymbol{BC} \to D, \boldsymbol{BCF} \to E\}$ ✓

$\{B\} \subseteq \{B, C\} \subseteq \{B, C, F\}$

$\{\boldsymbol{B} \to A, \boldsymbol{BC} \to D, \boldsymbol{BCG} \to E, \boldsymbol{BCF} \to H\}$ ✗

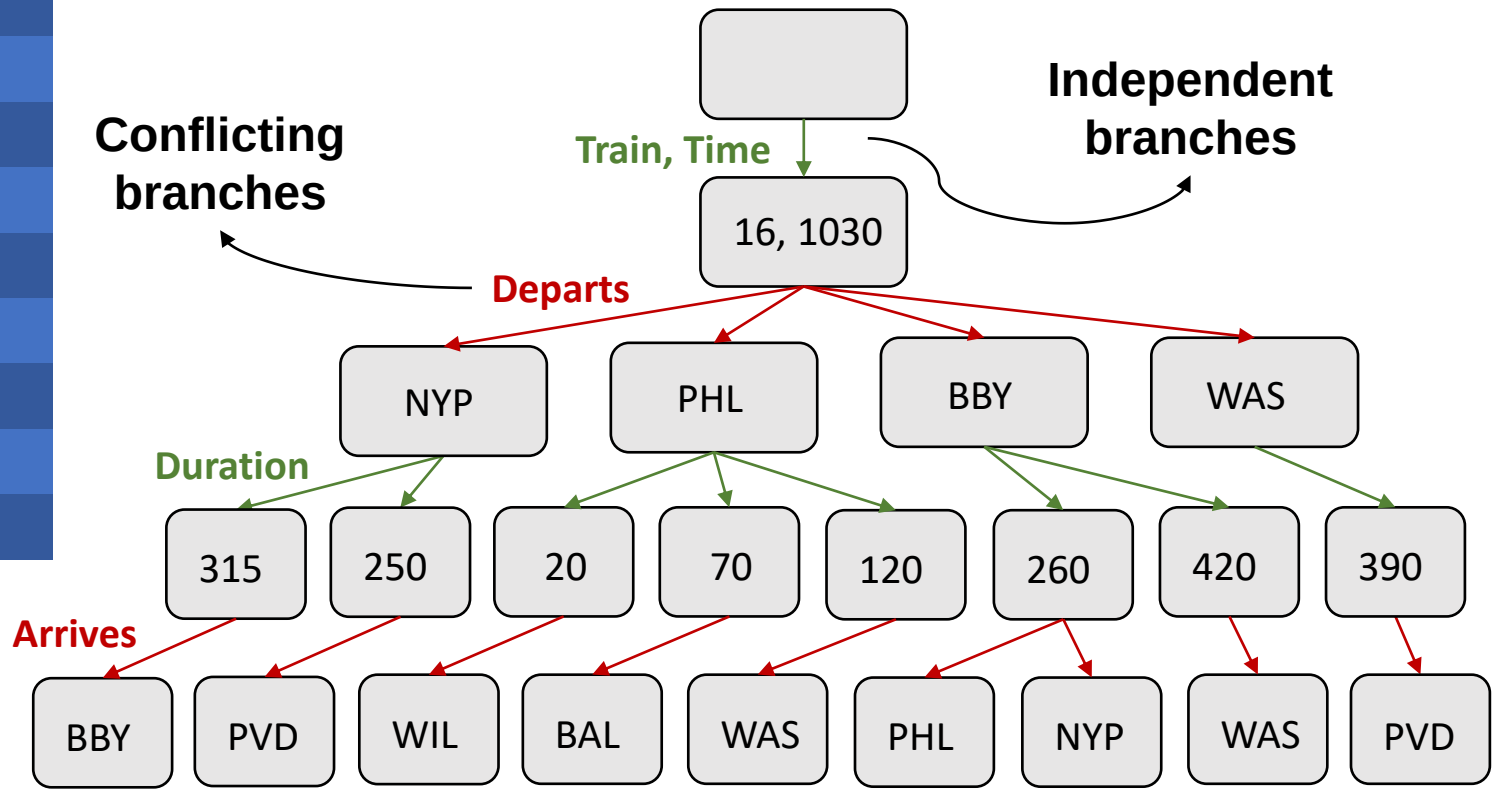
$\{B, C, G\} \not\subseteq \{B, C, F\}, \{B, C, F\} \not\subseteq \{B, C, G\}$

# Left-Hand Side Chain

Train Time → Departs

Train Time Duration → Arrives

| | Train | Departs | Arrives | Time | Duration |
|---|---|---|---|---|---|
| $f_1$ | 16 | NYP | BBY | 1030 | 315 |
| $f_2$ | 16 | NYP | PVD | 1030 | 250 |
| $f_3$ | 16 | PHL | WIL | 1030 | 20 |
| $f_4$ | 16 | PHL | BAL | 1030 | 70 |
| $f_5$ | 16 | PHL | WAS | 1030 | 120 |
| $f_6$ | 16 | BBY | PHL | 1030 | 260 |
| $f_7$ | 16 | BBY | NYP | 1030 | 260 |
| $f_8$ | 16 | BBY | WAS | 1030 | 420 |
| $f_9$ | 16 | WAS | PVD | 1030 | 390 |



**Efficiency:** $\sum_{a \in A} \text{Shapley}(A, v, a) = v(A)$

# Computational Complexity

| Measure | lhs chain | No lhs chain, tractable c-repair | other |
|---|---|---|---|
| drastic | PTIME | $\text{FP}^{\#\text{P}}$-complete | |
| #min-inconsistent | PTIME | | |
| #problematic tuples | PTIME | | |
| cardinality repair | PTIME | Open | NP-hard |
| #repairs | PTIME | $\text{FP}^{\#\text{P}}$-complete | |

$\{\boldsymbol{B} \rightarrow A, \boldsymbol{BC} \rightarrow D, \boldsymbol{BCF} \rightarrow E\}$ ✓

$\{B\} \subseteq \{BC\} \subseteq \{BCF\}$

$\{\boldsymbol{B} \rightarrow A, \boldsymbol{BC} \rightarrow D, \boldsymbol{BCG} \rightarrow E, \boldsymbol{BCF} \rightarrow H\}$ ✗

$\{BCG\} \not\subseteq \{BCF\}, \{BCF\} \not\subseteq \{BCG\}$

# Approximation Complexity

| Measure | lhs chain | No lhs chain, tractable c-repair | other |
|---|---|---|---|
| drastic | PTIME | FPRAS | |
| #min-inconsistent | PTIME | | |
| #problematic tuples | PTIME | | |
| cardinality repair | PTIME | FPRAS | No FPRAS |
| #repairs | PTIME | Open | |

Would imply an FPRAS for #MIS in a bipartite graph – long standing open problem

# Concluding Remarks

- Two situations where we wish to quantify the responsibility of tuples:
  - Query answering
  - Database inconsistency
- We treat the contribution from the viewpoint of game theory
- We investigated the computational complexity

# Thank you for listening!

## Questions?