TECHNION
Israel Institute of Technology

relationalAI

# The Shapley Value of Database Tuples

DBAI 2019

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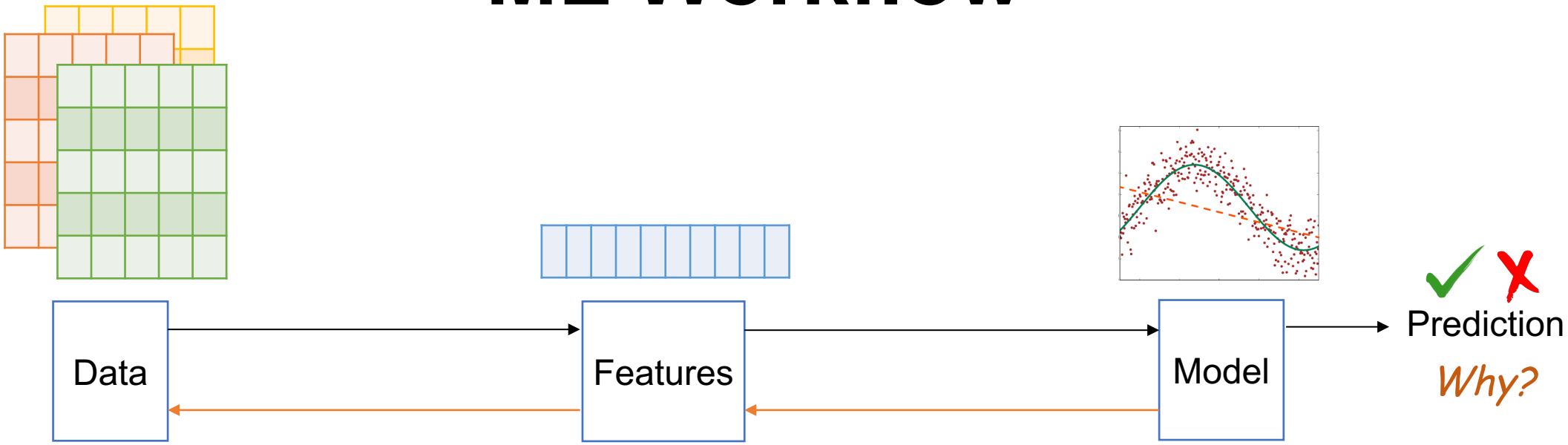
# ML Workflow




- Why provenance [Buneman+01,Green+07]
- Responsibility / causality / resilience [Meliou+10,15]

- Sensitivity analysis [Baehrens+10]
- Taylor decomposition [Montavon+17]
- Attention models [Ross+17]
- Shapley value [Lundberg&Lee17]

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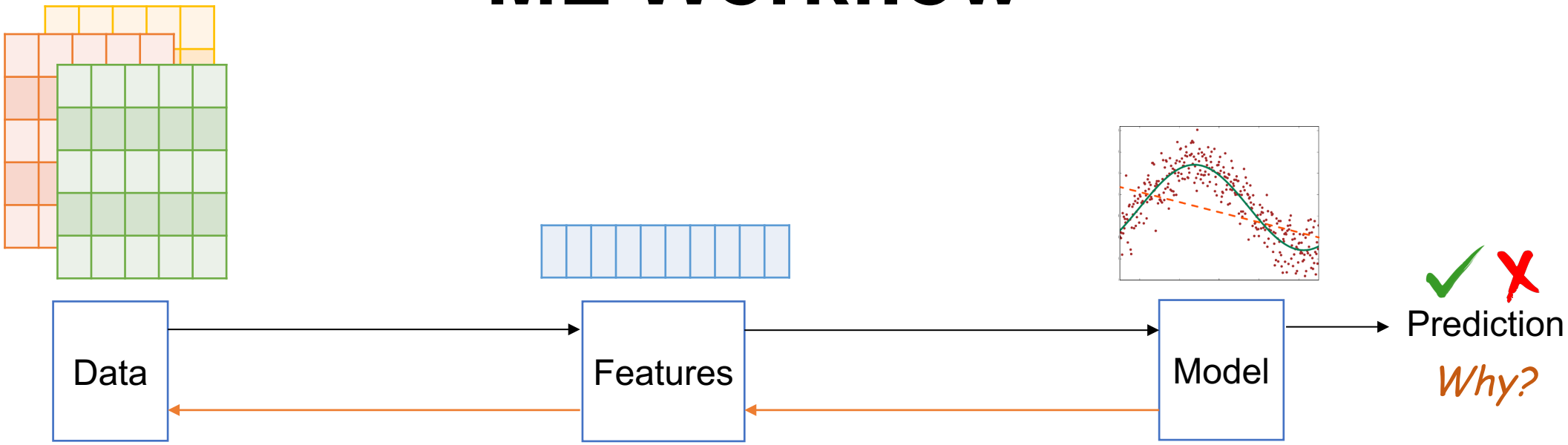


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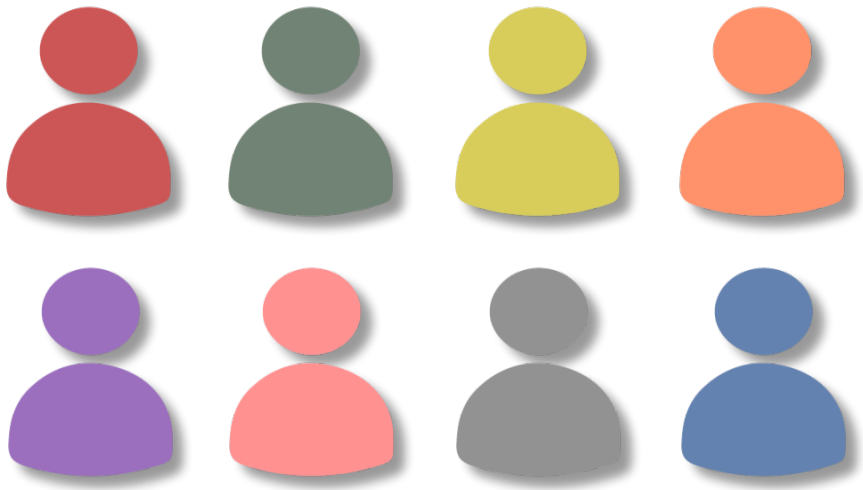
# Shapley Value

- A widely known profit-sharing formula in cooperative game theory
- Introduced by Lloyd Shapley in 1953
- Applied in various areas beyond cooperative game theory:
  - Pollution responsibility in environmental management
  - Influence measurement in social network analysis
  - Identifying candidate autism genes
  - Bargaining foundations in economics
  - Takeover corporate rights in law
  - Explanations in machine learning
- Received little attention in data management

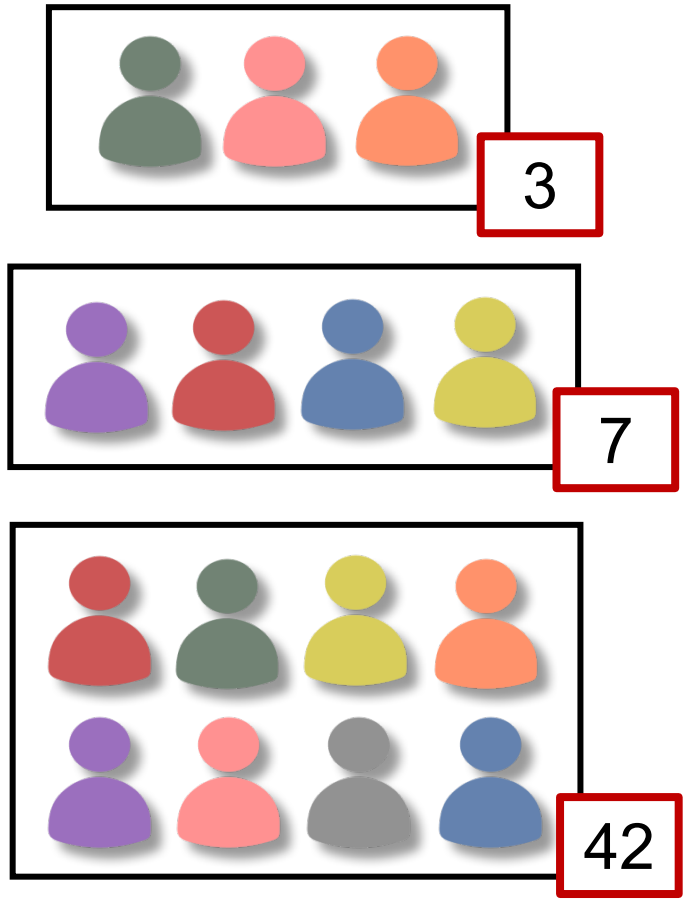
# Shapley Value

Set $A$ of players:



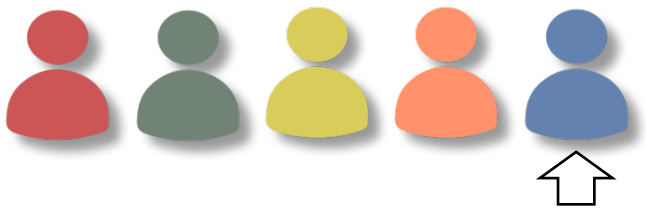
How to distribute the total wealth among the players?

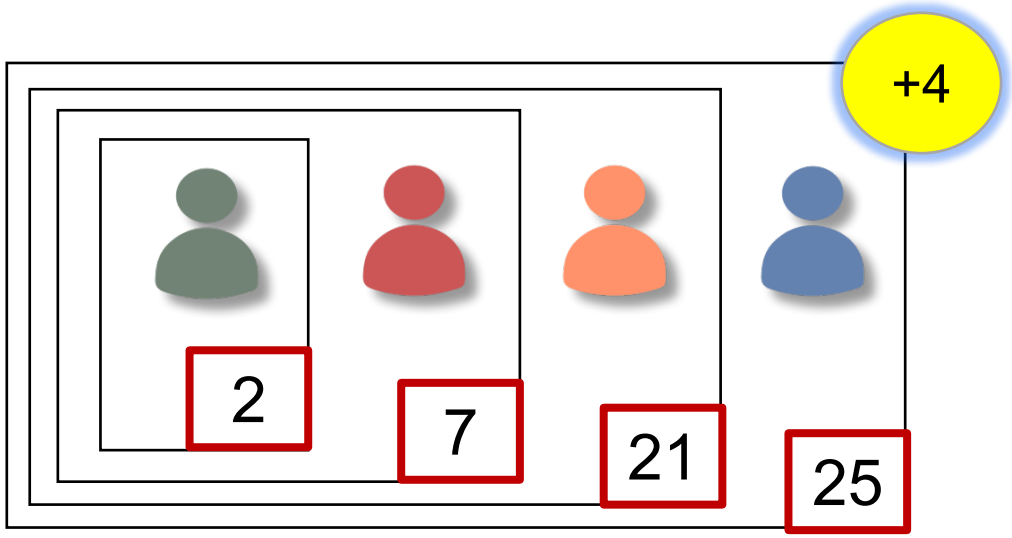
Wealth function $v: \mathcal{P}(A) \to \mathbb{R}$:



# Shapley Value

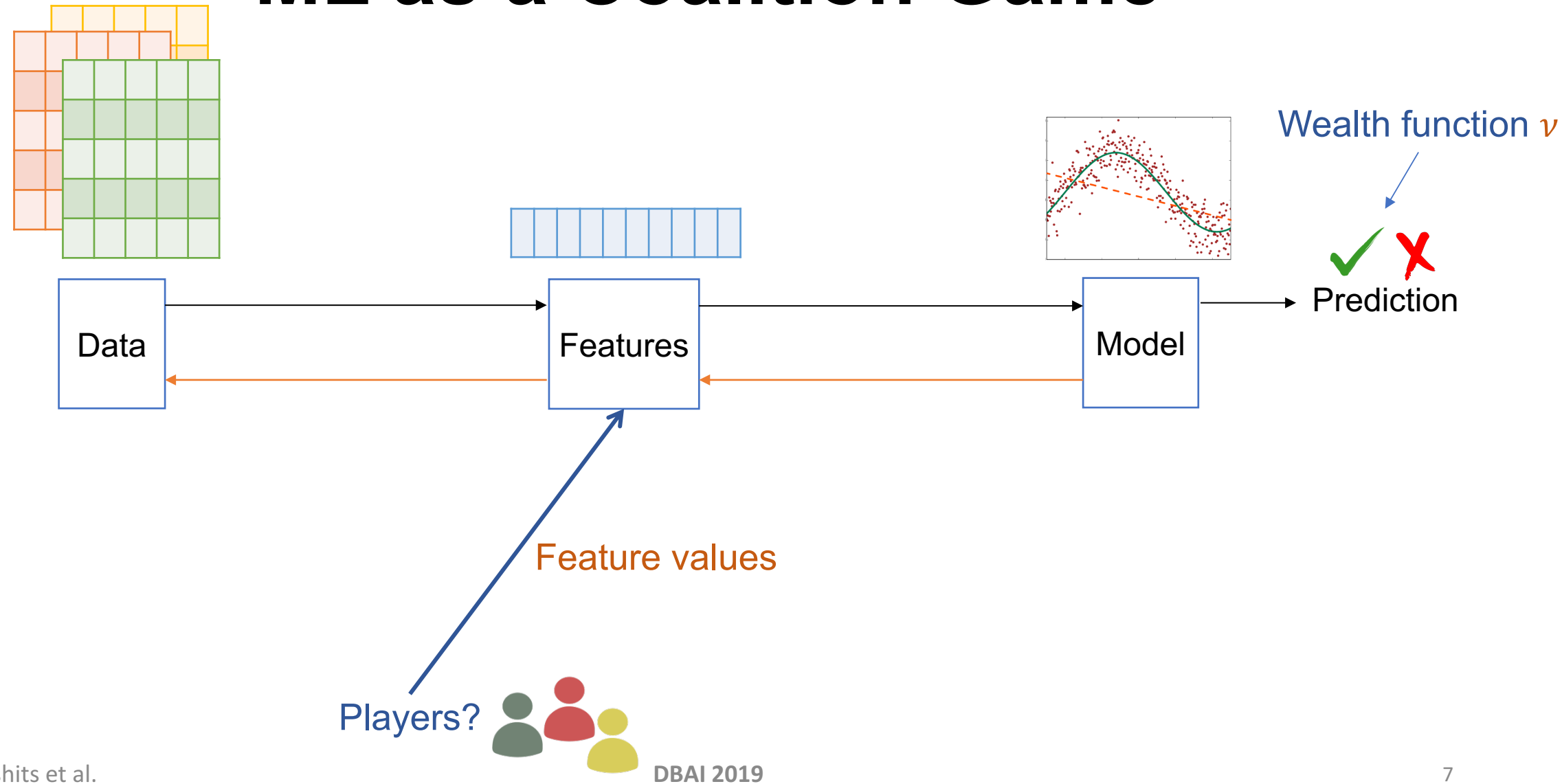
$$\text{Shapley}(A, v, a) = \sum_{B \subseteq A \setminus \{a\}} \frac{|B|!\,(|A| - |B| - 1)!}{|A|!} \big(v(B \cup \{a\}) - v(B)\big)$$



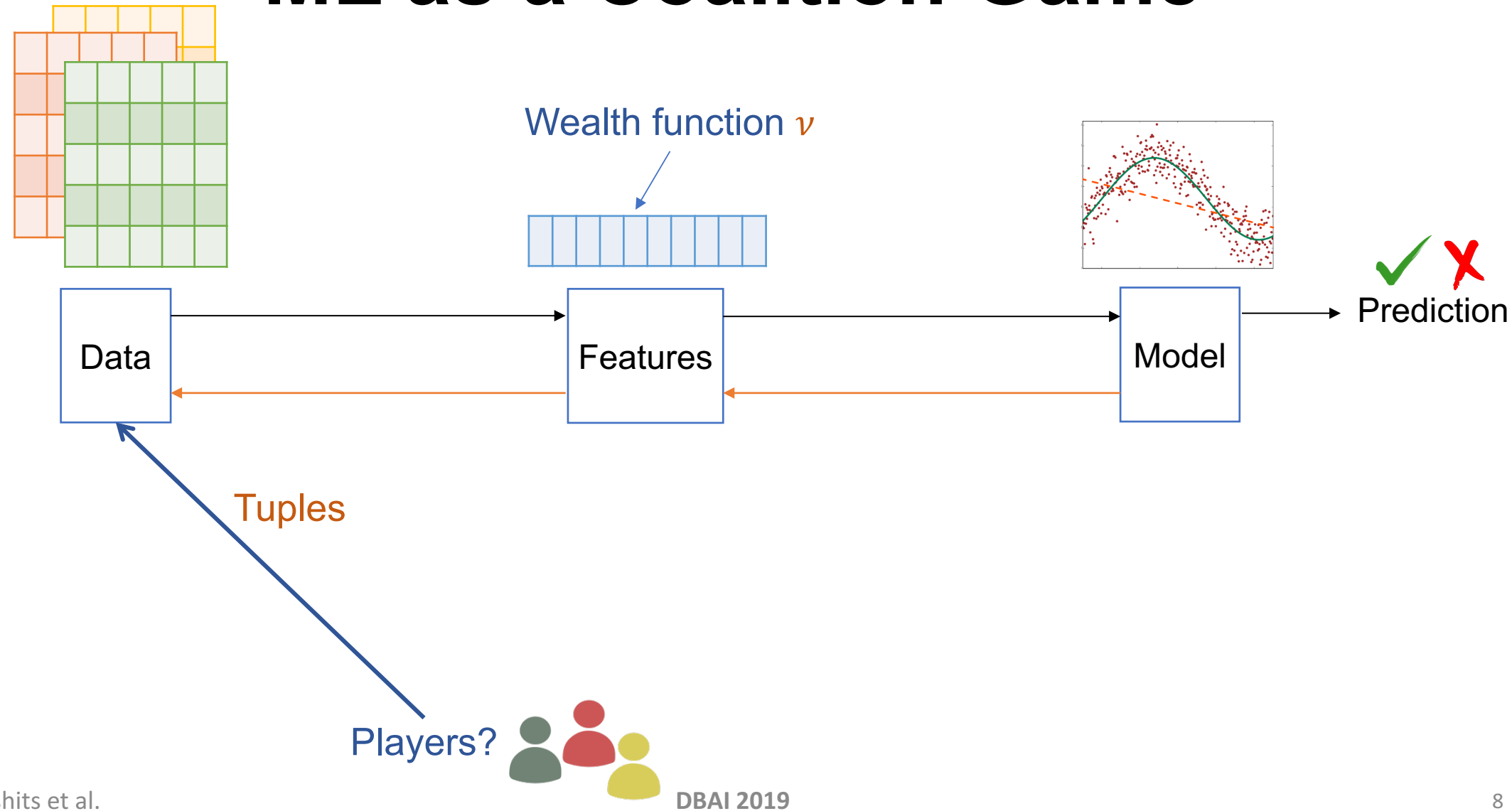



The Shapley value is the expected delta due to the addition in a random permutation

# ML as a Coalition Game



# ML as a Coalition Game



# Shapley Value for Database Queries

- **Goal:** quantify the contribution of tuples to query results

| AUTHOR | |
|---|---|
| **Name** | **Affiliation** |
| Alice | UCLA |
| Bob | NYU |
| Cathy | UCSD |
| David | MIT |
| Ellen | UCSD |

Players

| INSTITUTE | |
|---|---|
| **Name** | **STATE** |
| UCLA | CA |
| UCSD | CA |
| NYU | NY |
| MIT | MA |

| PUBLICAION | |
|---|---|
| **Author** | **Paper** |
| Alice | A |
| Alice | B |
| Bob | C |
| Cathy | C |
| Cathy | D |
| David | C |

| CITATIONS | |
|---|---|
| **PAPER** | **CITS** |
| A | 18 |
| B | 2 |
| C | 8 |
| D | 12 |

$$SV(Alice) = 20$$
$$SV(Cathy) = 14.67$$
$$SV(Bob) = 2.67$$
$$SV(David) = 2.67$$
$$SV(Ellen) = 0$$

$$q(z,w) :- AUTHOR(x,y), PUBLICATION(x,z), CITATIONS(z,w)$$

$$\mathrm{SUM}_w\langle q(z,w)\rangle$$

← Wealth function

# Boolean Conjunctive Queries

**Theorem (Dichotomy for Conjunctive Queries)**

Let $q$ be a Boolean CQ without self-joins.

- If $q$ is hierarchical, then $\mathrm{Shapley}(D, q, f)$ can be computed in **PTIME**.
- Otherwise, the problem is $FP^{\#P}$**-complete**.

- A CQ $q$ is **hierarchical** if for every two variables $x$ and $y$:
  - $Atoms(x) \subseteq Atoms(y)$ or
  - $Atoms(y) \subseteq Atoms(x)$ or
  - $Atoms(x) \cap Atoms(y) = \emptyset$

$q_1() :- R(x, y), S(x, z)$ ✓

$Atoms(x) = \{R(x, y), S(x, z)\}$
$Atoms(y) = \{R(x, y)\}$
$Atoms(z) = \{S(x, z)\}$

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$q_2() :- R(x), S(x, y), T(y)$ ✗

$Atoms(x) = \{R(x), S(x, y)\}$
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- Same classification criteria as for query answering over probabilistic databases [Dalvi and Suciu 2004]
- Can be easily extended to general CQs

# Aggregate Queries

- The Shapley value is a probabilistic expectation
- Using the linearity of expectation, we extend our dichotomy to SUM

| **Theorem (Dichotomy for SUM)** |
| --- |
| Let $\alpha$ be an arbitrary summation over a CQ $q$ without self-joins.<br>○ If $q$ is hierarchical, then $\mathrm{Shapley}(D, \alpha, f)$ can be computed in **PTIME**.<br>○ Otherwise, the problem is $FP^{\#P}$**-complete**. |

$$q(z,w) :- AUTHOR(x,y), PUBLICATION(x,z), CITATIONS(z,w)$$

$$\mathrm{SUM}_w\langle q(z,w)\rangle$$

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$$q(z,w) :- AUTHOR(x,y), PUBLICATION(x,z), CITATIONS(z,w), INST(y,'CA')$$

$$\mathrm{SUM}_w\langle q(z,w)\rangle$$

# Aggregate Queries

- Our hardness result applies to any non-constant numerical query

**Theorem (Hardness for Numerical Queries)**

Let $\alpha$ be a non-constant numerical query over a CQ $q$ without self-joins. If $q$ is non-hierarchical, then computing $\mathrm{Shapley}(D, \alpha, f)$ is $FP^{\#P}$**-complete**.

$$q(z,w) :- AUTHOR(x,y), PUBLICATION(x,z), CITATIONS(z,w), INST(y,'CA')$$

$$\mathrm{MAX}_{\mathrm{w}}\langle q(z,w)\rangle$$
$$\mathrm{MIN}_{\mathrm{w}}\langle q(z,w)\rangle$$
$$\mathrm{AVG}_{\mathrm{w}}\langle q(z,w)\rangle$$

# Approximation

- Computing the Shapley value is often hard
- But the picture is far more positive when allowing approximation

**Theorem**

For every fixed BCQ, the Shapley value has a multiplicative fully-polynomial randomized approximation scheme (FPRAS).

$$\Pr\left[\frac{f(x)}{1+\epsilon} \leq A(x,\epsilon,\delta) \leq (1+\epsilon)f(x)\right] \geq 1-\delta$$

- Generalizes to unions of CQs and summations over CQs

# Concluding Remarks

- We investigated the problem of quantifying the contribution of database tuples to query results via the Shapley value
- Related measures:
  - Causal responsibility [Meliou et al. 2010]
    - Not extendable to aggregate queries
  - Causal effect [Salimi et al. 2016]
    - We show that it coincides with the Banzhaf Power Index [Banzhaf 1965]
    - Our complexity results extend to this measure

# Concluding Remarks

- Open problems:
  - CQs with self joins ($q() :- R(x, y), R(y, z)$)
  - Extend our understanding of aggregates over CQs
  - Approximation for non-monotonic queries
  - Detecting sets of tuples with a high Shapley group value

# Thank you for listening!

## Questions?